Cuaderno de notas de trabajo

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Cuaderno 25

Principio Variacional.

$$(1)\quad \boxed{\delta \int_A^B \frac{M_{ij}\, dx^i\, dx^j}{\sqrt{N_{pq}\, dx^p\, dx^q}} = 0}$$

1) El escenario es un espacio de cuatro dimensiones; las coordenadas x^1, x^2, x^3, x^4.

$$dx^i = (dx^1, dx^2, dx^3, dx^4).$$

2) M_{ij} y N_{pq} son dos tensores doblemente covariantes.

3) A y B son dos puntos del espacio de 4 dimensiones,

$A(x^1_A, x^2_A, x^3_A, x^4_A)$;

$B(x^1_B, x^2_B, x^3_B, x^4_B)$.

~~4) No es necesario~~

4) La métrica del espacio de 4 dimensiones es:

$$ds^2 = g_{ij}\, dx^i\, dx^j.$$

Notación:

(2) $$\boxed{dQ = \frac{M_{ij}\, dx^i\, dx^j}{\sqrt{N_{pq}\, dx^p\, dx^q}}}$$

El cociente dQ

Principio variacional

$$\delta \int_A^B dQ = 0$$

(1)

El numerador del cociente dQ.

(3) $$\boxed{d\nu^2 = M_{ij}\, dx^i\, dx^j}$$

numerador

El denominador del cociente dQ

(4) $$\boxed{d\tau = \sqrt{N_{pq}\, dx^p\, dx^q}}$$

denominador

(5) $$\boxed{d\tau^2 = N_{pq}\, dx^p dx^q}$$ Subradical

(2) $$\boxed{dQ = \frac{d\nu^2}{d\tau}}$$

(2) $$\boxed{dQ = \frac{d\nu^2}{\sqrt{d\tau^2}}}$$

La variación del numerador $\delta(d\nu^2)$

$$\boxed{\delta(d\nu^2)}$$

Según (3) $d\nu^2 = M_{ij}\, dx^i dx^j$

$$\delta(d\nu^2) = \left\{ \begin{array}{l} \dfrac{\partial M_{ij}}{\partial x^k}\, dx^i dx^j \delta x^k \\ + M_{ij}\, d(\delta x^i) dx^j + M_{ij}\, dx^i d(\delta x^j) \end{array} \right\}$$

(6) $$\boxed{\delta(d\nu^2) = \left\{ \frac{\partial M_{ij}}{\partial x^k} dx^i dx^j \delta x^k + M_{kj}\, dx^j\, d(\delta x^k) + M_{ik}\, dx^i\, d(\delta x^k) \right\}}$$

La variación del denominador

$\boxed{\delta(d\tau)}$

(4) $$\boxed{d\tau = \sqrt{N_{pq}\, dx^p dx^q}}$$

$$\delta(d\tau) = \frac{\frac{\partial N_{pq}}{\partial x^k} dx^p dx^q \delta x^k + N_{pq}\, d(\delta x^p)\, dx^q + N_{pq}\, dx^p d(\delta x^q)}{2\sqrt{N_{pq}\, dx^p dx^q}}$$

(7) $$\boxed{\delta(d\tau) = \frac{\frac{\partial N_{pq}}{\partial x^k} dx^p dx^q \delta x^k + N_{kq}\, dx^q d(\delta x^k) + N_{pk}\, dx^p d(\delta x^k)}{2\, d\tau}}$$

La variación del cociente dQ

$$(2) \quad dQ = \frac{d\nu^2}{d\tau} \quad \text{(página 3)}$$

$$\delta(dQ) = \frac{\delta(d\nu^2)}{d\tau} - \frac{d\nu^2\, \delta(d\tau)}{d\tau^2}$$

$$(8) \quad \delta(dQ) = \left[\frac{\partial M_{ij}}{\partial x^k} \frac{dx^i}{d\tau} dx^j \delta x^k + M_{kj} \frac{dx^j}{d\tau} d(\delta x^k) + M_{ik} \frac{dx^i}{d\tau} d(\delta x^k) - \frac{1}{2} M_{ij} \frac{dx^i}{d\tau} \frac{dx^j}{d\tau} \left\langle \frac{\partial N_{pq}}{\partial x^k} \frac{dx^p}{d\tau} dx^q \delta x^k + N_{kq} \frac{dx^q}{d\tau} d(\delta x^k) + N_{pk} \frac{dx^p}{d\tau} d(\delta x^k) \right\rangle \right.$$

$$I_1 = \int_A^B \frac{\partial M_{ij}}{\partial x^k} \frac{dx^i}{d\tau} \frac{dx^j}{d\tau} \delta x^k \, d\tau \; ; \tag{9.1}$$

$$I_2 = \int_A^B M_{kj} \frac{dx^j}{d\tau} d(\delta x^k) ; \tag{9.2}$$

$$I_3 = \int_A^B M_{ik} \frac{dx^i}{d\tau} d(\delta x^k) ; \tag{9.3}$$

$$I_4 = -\frac{1}{2} \int_A^B M_{ij} \frac{\partial N_{pq}}{\partial x^k} \frac{dx^i}{d\tau} \frac{dx^j}{d\tau} \frac{dx^p}{d\tau} \frac{dx^q}{d\tau} \delta x^k \, d\tau ; \tag{9.4}$$

$$I_5 = -\frac{1}{2} \int_A^B M_{ij} N_{kq} \frac{dx^i}{d\tau} \frac{dx^j}{d\tau} \frac{dx^q}{d\tau} d(\delta x^k) ; \tag{9.5}$$

$$I_6 = -\frac{1}{2} \int_A^B M_{ij} N_{pk} \frac{dx^i}{d\tau} \frac{dx^j}{d\tau} \frac{dx^p}{d\tau} d(\delta x^k) . \tag{9.6}$$

Transformación de I_2

9.2) $$\boxed{I_2 = \int_A^B M_{kj} \frac{dx^j}{d\tau} \, d(\delta x^k).}$$

$$I_2 = -\int_A^B \delta x^k \left\{ \frac{\partial M_{kj}}{\partial x^i} \frac{dx^i}{d\tau} \frac{dx^j}{d\tau} + M_{kj} \frac{d^2 x^j}{d\tau^2} \right\} d\tau$$

$$\boxed{I_2 = -\int_A^B \left[\frac{\partial M_{kj}}{\partial x^i} \frac{dx^i}{d\tau} \frac{dx^j}{d\tau} + M_{kj} \frac{d^2 x^j}{d\tau^2} \right] \delta x^k \, d\tau}$$

Transformación de I_3

9.3) $$\boxed{I_3 = \int_A^B M_{ik} \frac{dx^i}{d\tau} d(\delta x^k).}$$

10.3) $$\boxed{I_3 = -\int_A^B \left[\frac{\partial M_{ik}}{\partial x^j}\frac{dx^i}{d\tau}\frac{dx^j}{d\tau} + M_{ik}\frac{d^2x^i}{d\tau^2}\right]\delta x^k \, d\tau}$$

Transformación de I_5

$$\boxed{I_5 = -\frac{1}{2}\int_A^B M_{ij} N_{kq} \frac{dx^i}{d\tau}\frac{dx^j}{d\tau}\frac{dx^q}{d\tau}\, d(\delta x^k)}$$

$$\boxed{I_5 = +\frac{1}{2}\int_A^B \left[\left\langle \frac{\partial M_{ij}}{\partial x^p} N_{kq} + M_{ij}\frac{\partial N_{kq}}{\partial x^p} \right\rangle \frac{dx^i}{d\tau}\frac{dx^j}{d\tau}\frac{dx^p}{d\tau}\frac{dx^q}{d\tau} + M_{ij} N_{kq}\frac{d^2x^i}{d\tau^2}\frac{dx^j}{d\tau}\frac{dx^q}{d\tau} + M_{ij} N_{kq}\frac{dx^i}{d\tau}\frac{d^2x^j}{d\tau}\frac{dx^q}{d\tau} + M_{ij} N_{kq}\frac{dx^i}{d\tau}\frac{dx^j}{d\tau}\frac{d^2x^q}{d\tau^2} \right] \delta x^k\, d\tau}$$

Transformación de I_6.

$$(9.6)\quad \boxed{I_6 = -\frac{1}{2}\int_A^B M_{ij} N_{pk} \frac{dx^i}{d\tau}\frac{dx^j}{d\tau}\frac{dx^p}{d\tau}\, d(\delta x^k).}$$

$$\boxed{I_6 = +\frac{1}{2}\int_A^B \left[\left\langle \frac{\partial M_{ij}}{\partial x^q} N_{pk} + M_{ij}\frac{\partial N_{pk}}{\partial x^q} \right\rangle \frac{dx^i}{d\tau}\frac{dx^j}{d\tau}\frac{dx^p}{d\tau}\frac{dx^q}{d\tau} + M_{ij} N_{pk} \frac{d^2x^i}{d\tau}\frac{dx^j}{d\tau}\frac{dx^p}{d\tau} + M_{ij} N_{pk}\frac{dx^i}{d\tau}\frac{d^2x^j}{d\tau}\frac{dx^p}{d\tau} + M_{ij} N_{pk}\frac{dx^i}{d\tau}\frac{dx^j}{d\tau}\frac{d^2x^p}{d\tau^2} \right] \delta x^k\, d\tau}$$

Ecuaciones diferenciales de las extremales.

$$\frac{\partial M_{ij}}{\partial x^k}\frac{dx^i}{d\tau}\frac{dx^j}{d\tau} - \frac{\partial M_{kj}}{\partial x^i}\frac{dx^i}{d\tau}\frac{dx^j}{d\tau} - M_{kj}\frac{d^2x^j}{d\tau^2}$$

$$- \frac{\partial M_{ik}}{\partial x^j}\frac{dx^i}{d\tau}\frac{dx^j}{d\tau} - M_{ik}\frac{d^2x^i}{d\tau^2}$$

$$-\frac{1}{2}M_{ij}\frac{\partial N_{pq}}{\partial x^k}\frac{dx^i}{d\tau}\frac{dx^j}{d\tau}\frac{dx^p}{d\tau}\frac{dx^q}{d\tau}$$

$$0 = \quad +\frac{1}{2}\left(\frac{\partial M_{ij}}{\partial x^p}N_{kq} + M_{ij}\frac{\partial N_{kq}}{\partial x^p}\right)\frac{dx^i}{d\tau}\frac{dx^j}{d\tau}\frac{dx^p}{d\tau}\frac{dx^q}{d\tau}$$

$$+\frac{1}{2}M_{ij}N_{kq}\frac{d^2x^i}{d\tau^2}\frac{dx^j}{d\tau}\frac{dx^q}{d\tau}$$

$$+\frac{1}{2}M_{ij}N_{kq}\frac{dx^i}{d\tau}\frac{d^2x^j}{d\tau^2}\frac{dx^q}{d\tau}$$

$$+\frac{1}{2}M_{ij}N_{kq}\frac{dx^i}{d\tau}\frac{dx^j}{d\tau}\frac{d^2x^q}{d\tau^2}$$

$$+\frac{1}{2}\left(\frac{\partial M_{ij}}{\partial x^q}N_{pk} + M_{ij}\frac{\partial N_{pk}}{\partial x^q}\right)\frac{dx^i}{d\tau}\frac{dx^j}{d\tau}\frac{dx^p}{d\tau}\frac{dx^q}{d\tau}$$

$$+\frac{1}{2}M_{ij}N_{pk}\frac{d^2x^i}{d\tau^2}\frac{dx^j}{d\tau}\frac{dx^p}{d\tau}$$

$$+\frac{1}{2}M_{ij}N_{pk}\frac{dx^i}{d\tau}\frac{d^2x^j}{d\tau^2}\frac{dx^p}{d\tau}$$

$$+\frac{1}{2}M_{ij}N_{pk}\frac{dx^i}{d\tau}\frac{dx^j}{d\tau}\frac{d^2x^p}{d\tau^2}$$

$$\cancel{\frac{dx^i}{d\tau} = \frac{dx^j}{ds}}$$

$$\frac{dx^i}{ds} = \frac{dx^i}{d\tau}\frac{d\tau}{ds}$$

$$\frac{d^2x^i}{ds^2} = \frac{d^2x^i}{d\tau^2}\left(\frac{d\tau}{ds}\right)^2 + \frac{dx^i}{d\tau}\frac{d^2\tau}{ds^2}$$

$\alpha 1)$
$$\boxed{\frac{d^2x^i}{d\tau^2} = \frac{\frac{d^2x^i}{ds^2}}{\left(\frac{d\tau}{ds}\right)^2} - \frac{dx^i}{d\tau}\frac{\frac{d^2\tau}{ds^2}}{\left(\frac{d\tau}{ds}\right)^2}}$$

(4)
$$\boxed{d\tau = \sqrt{N_{pq}\, dx^p\, dx^q}}$$

pág. 2.

$\alpha 2)$
$$\boxed{\frac{d\tau}{ds} = \sqrt{N_{pq}\frac{dx^p}{ds}\frac{dx^q}{ds}}}$$

$$\frac{d^2\tau}{ds^2} = \frac{\frac{\partial N_{pq}}{\partial x^r}\frac{dx^p}{ds}\frac{dx^q}{ds}\frac{dx^r}{ds} + N_{pq}\frac{d^2x^p}{ds^2}\frac{dx^q}{ds} + N_{pq}\frac{dx^p}{ds}\frac{d^2x^q}{ds^2}}{2\sqrt{N_{pq}\frac{dx^p}{ds}\frac{dx^q}{ds}}}$$

$$\frac{d^2\tau}{ds^2} = \frac{\partial N_{pq}}{\partial x^r}\frac{dx^p}{ds}\frac{dx^q}{ds}\frac{dx^r}{ds} + N_{pq}\frac{d^2x^p}{ds^2}\frac{dx^q}{ds} + N_{pq}\frac{dx^p}{ds}\frac{d^2x^q}{ds^2}$$

α3) $$\boxed{\frac{d^2\tau}{ds^2} = \frac{\dfrac{\partial N_{pq}}{\partial x^r}\dfrac{dx^p}{ds}\dfrac{dx^q}{ds}\dfrac{dx^r}{ds} + N_{pq}\dfrac{d^2x^p}{ds^2}\dfrac{dx^q}{ds} + N_{pq}\dfrac{dx^p}{ds}\dfrac{d^2x^q}{ds^2}}{2\sqrt{N_{pq}\dfrac{dx^p}{ds}\dfrac{dx^q}{ds}}}}$$

α4) $$\boxed{\frac{d^2x^i}{ds^2} = \left(\frac{\partial h_{ip}}{\partial x^q} - \frac{\partial h_{pq}}{\partial x^i}\right)\frac{dx^p}{ds}\frac{dx^q}{ds}}$$

$$\frac{d^2x^i}{d\tau^2} = \left[\left(\frac{\partial h_{ip}}{\partial x^q} - \frac{\partial h_{pq}}{\partial x^i}\right)\frac{dx^p}{d\tau}\frac{dx^q}{d\tau}\right.$$

$$- \frac{1}{2}\frac{dx^i}{d\tau}\left[\frac{\partial N_{pq}}{\partial x^r}\frac{dx^p}{d\tau}\frac{dx^q}{d\tau}\frac{dx^r}{d\tau}\right.$$

(4) $$\boxed{d\tau = \sqrt{N_{pq}\, dx^p\, dx^q}}$$

α 2) $$\boxed{\frac{d\tau}{ds} = \sqrt{N_{pq}\frac{dx^p}{ds}\frac{dx^q}{ds}}}$$

α 3) $$\boxed{\frac{d^2\tau}{ds^2} = \frac{\frac{\partial N_{pq}}{\partial x^r}\frac{dx^p}{ds}\frac{dx^q}{ds}\frac{dx^r}{ds} + N_{pq}\frac{d^2x^p}{ds^2}\frac{dx^q}{ds} + N_{pq}\frac{dx^p}{ds}\frac{d^2x^q}{ds^2}}{2\sqrt{N_{pq}\frac{dx^p}{ds}\frac{dx^q}{ds}}}}$$

$$\frac{d^2\tau}{ds^2} = \frac{1}{2\left(\frac{d\tau}{ds}\right)}\left[\frac{\partial N_{pq}}{\partial x^r}\frac{dx^p}{ds}\frac{dx^q}{ds}\frac{dx^r}{ds} + N_{pq}\left(\frac{\partial h_{pr}}{\partial x^t} - \frac{\partial h_{rt}}{\partial x^p}\right)\frac{dx^q}{ds}\frac{dx^r}{ds}\frac{dx^t}{ds} + N_{pq}\left(\frac{\partial h_{qr}}{\partial x^t} - \frac{\partial h_{rt}}{\partial x^q}\right)\frac{dx^p}{ds}\frac{dx^r}{ds}\frac{dx^t}{ds}\right]$$

$$\frac{d^2\tau}{ds^2} = \frac{1}{2\left(\frac{d\tau}{ds}\right)}\left[\frac{\partial N_{pq}}{\partial x^r} + N_{pq}\left(\partial h_{rr}\right.\right]\frac{dx^p}{ds}\frac{dx^q}{ds}\frac{dx^r}{ds}$$

$$\frac{d^2\tau}{ds^2} = \frac{1}{2\left(\frac{d\tau}{ds}\right)} \left[\frac{\partial N_{pq}}{\partial x^r} + N_{tq}\left(\frac{\partial h_{tr}}{\partial x^p} - \frac{\partial h_{rp}}{\partial x^t}\right) + N_{pt}\left(\frac{\partial h_{tr}}{\partial x^q} - \frac{\partial h_{rq}}{\partial x^t}\right) \right] \frac{dx^p}{ds}\frac{dx^q}{ds}\frac{dx^r}{ds} \tag{α5}$$

$$\frac{\left(\frac{d^2\tau}{ds^2}\right)}{\left(\frac{d\tau}{ds}\right)^2} = \frac{1}{2} \left[\frac{\partial N_{pq}}{\partial x^r} + N_{tq}\left(\frac{\partial h_{tr}}{\partial x^p} - \frac{\partial h_{rp}}{\partial x^t}\right) + N_{pt}\left(\frac{\partial h_{tr}}{\partial x^q} - \frac{\partial h_{rq}}{\partial x^t}\right) \right] \frac{dx^p}{d\tau}\frac{dx^q}{d\tau}\frac{dx^r}{d\tau} \tag{α6}$$

Ecuaciones de las líneas de universo de Birkhoff con $d\tau$ como parámetro.

α 4 pág. 13) $$\frac{d^2x^i}{ds^2} = \left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i}\right)\frac{dx^j}{ds}\frac{dx^k}{ds}$$

$$\cancel{\frac{dx^i}{d\tau} = \frac{dx^i}{ds} \Big/ \frac{d\tau}{ds}}$$

$$\frac{dx^i}{d\tau} = \frac{\frac{dx^i}{ds}}{\frac{d\tau}{ds}}$$

$$\frac{d^2x^i}{d\tau^2} = \frac{\frac{d\tau}{ds}\frac{d^2x^i}{ds^2} - \frac{dx^i}{ds}\frac{d^2\tau}{ds^2}}{\left(\frac{d\tau}{ds}\right)^3}$$

$$\boxed{\frac{d^2x^i}{d\tau^2} = \left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i}\right)\frac{dx^j}{d\tau}\frac{dx^k}{d\tau} - \frac{\frac{d^2\tau}{ds^2}}{\left(\frac{d\tau}{ds}\right)^2}\frac{dx^i}{d\tau}}$$

4 pág. 2) $d\tau = \sqrt{N_{pq}\, dx^p dx^q}$

α2 pág 12) $\frac{d\tau}{ds} = \sqrt{N_{pq} \frac{dx^p}{ds} \frac{dx^q}{ds}}$

$$\frac{d^2\tau}{ds^2} = \frac{N_{pq} \frac{d^2x^p}{ds^2} \frac{dx^q}{ds} + N_{pq} \frac{dx^p}{ds} \frac{d^2x^q}{ds^2} + \frac{\partial N_{pq}}{\partial x^r} \frac{dx^p}{ds} \frac{dx^q}{ds} \frac{dx^r}{ds}}{2\sqrt{N_{pq} \frac{dx^p}{ds} \frac{dx^q}{ds}}}$$

$$\frac{d^2\tau}{ds^2} = \frac{1}{2} \left[N_{pq} \left(\frac{\partial h_{pa}}{\partial x^b} - \frac{\partial h_{ab}}{\partial x^p} \right) \frac{dx^a}{ds} \frac{dx^b}{ds} \frac{dx^q}{d\tau} + N_{pq} \left(\frac{\partial h_{qa}}{\partial x^b} - \frac{\partial h_{ab}}{\partial x^q} \right) \frac{dx^p}{d\tau} \frac{dx^a}{ds} \frac{dx^b}{ds} + \frac{\partial N_{pq}}{\partial x^r} \frac{dx^p}{ds} \frac{dx^q}{ds} \frac{dx^r}{d\tau} \right]$$

$$\frac{\frac{d^2\tau}{ds^2}}{\left(\frac{d\tau}{ds}\right)^2} = \frac{1}{2} \left[N_{pq} \left(\frac{\partial h_{pa}}{\partial x^b} - \frac{\partial h_{ab}}{\partial x^p} \right) \frac{dx^a}{d\tau} \frac{dx^b}{d\tau} \frac{dx^q}{d\tau} + N_{pq} \left(\frac{\partial h_{qa}}{\partial x^b} - \frac{\partial h_{ab}}{\partial x^q} \right) \frac{dx^a}{d\tau} \frac{dx^b}{d\tau} \frac{dx^p}{d\tau} + \frac{\partial N_{pq}}{\partial x^r} \frac{dx^p}{d\tau} \frac{dx^q}{d\tau} \frac{dx^r}{d\tau} \right]$$

$$\frac{\frac{d^2\tau}{ds^2}}{\left(\frac{d\tau}{ds}\right)^2} = \frac{1}{2}\left[\left\{(N_{pq} + N_{qp})\left(\frac{\partial h_{pa}}{\partial x^b} - \frac{\partial h_{ab}}{\partial x^p}\right) + \frac{\partial N_{ab}}{\partial x^q}\right\}\frac{dx^a}{d\tau}\frac{dx^b}{d\tau}\frac{dx^q}{d\tau}\right]$$

α 6 pág 15)

$$\boxed{\frac{\frac{\partial^2\tau}{ds^2}}{\left(\frac{d\tau}{ds}\right)^2} = \left[N_{pq}\left(\frac{\partial h_{pa}}{\partial x^b} - \frac{\partial h_{ab}}{\partial x^p}\right) + \frac{1}{2}\frac{\partial N_{ab}}{\partial x^q}\right]\frac{dx^a}{d\tau}\frac{dx^b}{d\tau}\frac{dx^q}{d\tau}}$$

①

$$\frac{d^2x^i}{ds^2} = \Delta^{im}\left[\frac{\partial h_{mj}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^m}\right]\frac{dx^j}{ds}\frac{dx^k}{ds}.$$

$$\frac{d^2t}{ds^2} = \left[\frac{\partial h_{1j}}{\partial x^k} - \frac{\partial h_{kj}}{\partial t}\right]\frac{dx^j}{ds}\frac{dx^k}{ds}$$

$$\frac{d^2t}{ds^2} = \cancel{+\left[\frac{\partial h_{11}}{\partial t} - \frac{\partial h_{11}}{\partial t}\right]\frac{dt}{ds}\frac{dt}{ds}}$$
$$+\left[\frac{\partial h_{11}}{\partial x^\alpha} - \frac{\partial h_{\alpha 1}}{\partial t}\right]\frac{dt}{ds}\frac{dx^\alpha}{ds}$$
$$\cancel{+\left[\frac{\partial h_{1\alpha}}{\partial t} - \frac{\partial h_{1\alpha}}{\partial t}\right]\frac{dx^\alpha}{ds}\frac{dt}{ds}}$$
$$+\left[\frac{\partial h_{1\alpha}}{\partial x^\beta} - \frac{\partial h_{\beta\alpha}}{\partial t}\right]\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds}$$

$$\frac{d^2x^i}{ds^2} = -\left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i}\right)\frac{dx^j}{ds}\frac{dx^k}{ds} \qquad i = 2, 3, 4.$$

$$\boxed{\frac{d^2x}{ds^2} = -\left(\frac{\partial h_{2j}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x}\right)\frac{dx^j}{ds}\frac{dx^k}{ds}} \qquad \boxed{i=2}$$

$$\boxed{\frac{d^2t}{ds^2} = \left(\frac{\partial h_{1j}}{\partial x^k} - \frac{\partial h_{jk}}{\partial t}\right)\frac{dx^j}{ds}\frac{dx^k}{ds}.}$$

$$\frac{dx}{dt} = \cancel{\frac{dx}{ds}}\ \frac{\frac{dx}{ds}}{\frac{dt}{ds}} = \frac{x'}{t'}$$

$$\dot{x} = \frac{x'}{t'}$$

$$\ddot{x} = \frac{t'x'' - x't''}{t'^3} = \frac{\frac{d}{ds}(\dot{x})}{\frac{dt}{ds}}$$

$$\boxed{\ddot{x} = \frac{x''}{t'^2} - \dot{x}\,\frac{t''}{t'^2}}$$

$$\ddot{x} = \left[\left(\frac{\partial h_{jk}}{\partial x} - \frac{\partial h_{2j}}{\partial x^k} \right) \dot{x}^j \dot{x}^k + \left(\frac{\partial h_{jk}}{\partial t} - \frac{\partial h_{1j}}{\partial x^k} \right) \dot{x}^j \dot{x}^k \dot{x} \right]$$

$$\ddot{x} = \left[\left(\frac{\partial h_{11}}{\partial x} - \frac{\partial h_{21}}{\partial t} \right) + \left(\frac{\partial h_{1\beta}}{\partial x} - \frac{\partial h_{21}}{\partial x^\beta} \right) \dot{x}^\beta + \left(\frac{\partial h_{\beta 1}}{\partial x} - \frac{\partial h_{2\beta}}{\partial t} \right) \dot{x}^\beta + \left(\frac{\partial h_{\beta\gamma}}{\partial x} - \frac{\partial h_{2\beta}}{\partial x^\gamma} \right) \dot{x}^\beta \dot{x}^\gamma \right.$$

$$+ \cancel{\left(\frac{\partial h_{11}}{\partial t} - \frac{\partial h_{11}}{\partial t} \right)} + \left(\frac{\partial h_{1\beta}}{\partial t} - \frac{\partial h_{11}}{\partial x^\beta} \right) \dot{x} \dot{x}^\beta + \cancel{\left(\frac{\partial h_{\beta 1}}{\partial t} - \frac{\partial h_{1\beta}}{\partial t} \right) \dot{x}^\beta \dot{x}} + \left(\frac{\partial h_{\beta\gamma}}{\partial t} - \frac{\partial h_{1\beta}}{\partial x^\gamma} \right) \dot{x} \dot{x}^\beta \dot{x}^\gamma$$

$$
\begin{aligned}
\ddot{x} = \Big[& \left(\frac{\partial h_{11}}{\partial x} - \frac{\partial h_{21}}{\partial t} \right) \\
& + \left(2\frac{\partial h_{1\beta}}{\partial x} - \frac{\partial h_{2\beta}}{\partial t} - \frac{\partial h_{12}}{\partial x^{\beta}} \right) \dot{x}^{\beta} \\
& + \left(\frac{\partial h_{1\beta}}{\partial t} - \frac{\partial h_{11}}{\partial x^{\beta}} \right) \dot{x}\,\dot{x}^{\beta} \\
& + \left(\frac{\partial h_{\beta\gamma}}{\partial x} - \frac{\partial h_{2\beta}}{\partial x^{\gamma}} \right) \dot{x}^{\beta}\dot{x}^{\gamma} \\
& + \left(\frac{\partial h_{\beta\gamma}}{\partial t} - \frac{\partial h_{1\beta}}{\partial x^{\gamma}} \right) \dot{x}\,\dot{x}^{\beta}\dot{x}^{\gamma}
\end{aligned}
$$

$$
\ddot{x}^{\alpha} = \left[\left(\frac{\partial h_{11}}{\partial x^{\alpha}} - \frac{\partial h_{1\alpha}}{\partial t} \right) \right.
$$
$$
+ \left(2 \frac{\partial h_{1\beta}}{\partial x^{\alpha}} - \frac{\partial h_{1\alpha}}{\partial x^{\beta}} - \frac{\partial h_{\alpha\beta}}{\partial t} \right) \dot{x}^{\beta}
$$
$$
+ \left(\frac{\partial h_{\beta\gamma}}{\partial x^{\alpha}} - \frac{\partial h_{\alpha\beta}}{\partial x^{\gamma}} \right) \dot{x}^{\beta} \dot{x}^{\gamma}
$$
$$
+ \left(\frac{\partial h_{1\beta}}{\partial t} - \frac{\partial h_{11}}{\partial x^{\beta}} \right) \dot{x}^{\alpha} \dot{x}^{\beta}
$$
$$
\left. + \left(\frac{\partial h_{\beta\gamma}}{\partial t} - \frac{\partial h_{1\beta}}{\partial x^{\gamma}} \right) \dot{x}^{\alpha} \dot{x}^{\beta} \dot{x}^{\gamma} \right]
$$

$h_{11} = \Phi =$ potencial escalar

$(h_{12}, h_{13}, h_{14}) = \vec{P} =$ $\left\{ \begin{matrix} \text{potencial} \\ \text{vectorial} \end{matrix} \right\}$

$$
\begin{pmatrix} h_{22} & h_{23} & h_{24} \\ h_{32} & h_{33} & h_{34} \\ h_{42} & h_{43} & h_{44} \end{pmatrix} = T^{\alpha\beta} = \left\{ \begin{matrix} \text{potencial} \\ \text{tensorial} \end{matrix} \right\}
$$

$\vec{a}$ = vector aceleración

$$\vec{a} = (\ddot{x}, \ddot{y}, \ddot{z}) = (\ddot{x}^2, \ddot{x}^3, \ddot{x}^4)$$

$\vec{v}$ = vector velocidad

$$\vec{v} = (\dot{x}, \dot{y}, \dot{z}) = (\dot{x}^2, \dot{x}^3, \dot{x}^4)$$

$$\vec{a} = \Big[\nabla\Phi - \frac{\partial \vec{P}}{\partial t} + 2\nabla(\vec{P}\cdot\vec{v}) - (\vec{v}\cdot\nabla)\vec{P} - \frac{\partial T}{\partial t}\vec{v} + \nabla\{(T\vec{v})\cdot\vec{v}\} - (\vec{v}\cdot\nabla)(T\vec{v}) + \left(\frac{\partial \vec{P}}{\partial t}\cdot\vec{v}\right)\vec{v} - \{(\vec{v}\cdot\nabla)\Phi\}\vec{v} + \left\{\left(\frac{\partial T}{\partial t}\vec{v}\right)\cdot\vec{v}\right\}\vec{v} - \{(\vec{v}\cdot\nabla)(\vec{P}\cdot\vec{v})\}\vec{v} \Big]$$

$$\vec{a} = \left\{ \nabla \left[\Phi + 2(\vec{P}\cdot\vec{v}) + (T\vec{v})\cdot\vec{v} \right] - \frac{\partial \vec{P}}{\partial t} - (\vec{v}\cdot\nabla)\vec{P} - \frac{\partial T}{\partial t}\vec{v} - (\vec{v}\cdot\nabla)(T\vec{v}) + \left[-(\vec{v}\cdot\nabla)\Phi + \frac{\partial \vec{P}}{\partial t}\cdot\vec{v} - (\vec{v}\cdot\nabla)(\vec{P}\cdot\vec{v}) + \left\{\left(\frac{\partial T}{\partial t}\vec{v}\right)\cdot\vec{v}\right\} \right]\vec{v} \right\}$$

$$\boxed{\vec{a} = \left\{ \nabla \left[\Phi + 2(\vec{P}\cdot\vec{v}) + (T\vec{v})\cdot\vec{v} \right] - \frac{\partial \vec{P}}{\partial t} - (\vec{v}\cdot\nabla)\vec{P} - \frac{\partial T}{\partial t}\vec{v} - (\vec{v}\cdot\nabla)(T\vec{v}) + \left[-(\vec{v}\cdot\nabla)\Phi + \frac{\partial \vec{P}}{\partial t}\cdot\vec{v} - (\vec{v}\cdot\nabla)(\vec{P}\cdot\vec{v}) + \left\{\left(\frac{\partial T}{\partial t}\vec{v}\right)\cdot\vec{v}\right\} \right]\vec{v} \right\}}$$

Caso del Campo Central.

$$\vec{P} = 0$$

$$\begin{pmatrix} \Phi & 0 & 0 & 0 \\ 0 & \Phi & 0 & 0 \\ 0 & 0 & \Phi & 0 \\ 0 & 0 & 0 & \Phi \end{pmatrix}$$

$$T = \begin{pmatrix} \Phi & 0 & 0 \\ 0 & \Phi & 0 \\ 0 & 0 & \Phi \end{pmatrix}$$

$$T\vec{v} = \Phi \vec{v} \qquad \frac{\partial \vec{P}}{\partial t} = 0 \qquad \frac{\partial \Phi}{\partial t} = 0$$

$$(T\vec{v}) \cdot \vec{v} = \Phi v^2$$

$$\vec{a} = \Big[\nabla \left[\Phi + \Phi v^2 \right] - (\vec{v} \cdot \nabla)(\Phi \vec{v}) - \left\{ (\vec{v} \cdot \nabla) \Phi \right\} \vec{v} \, \cancel{Am} \Big]$$

$$\vec{a} = (1+v^2)\,\nabla\Phi - 2\left\{(\vec{v}\cdot\nabla)\Phi\right\}\vec{v}$$

$$\Phi = \frac{M}{r} \qquad \nabla\Phi = -\frac{M\vec{r}}{r^3}$$

$$\vec{a} = -(1+v^2)\frac{M\vec{r}}{r^3} + 2\left(\frac{\vec{v}\cdot\vec{r}}{r^3}\right)M\vec{v}$$

$$\ddot{x} = +\frac{M}{r^3}\left[-x - xv^2 + 2r\dot{r}\dot{x}\right]$$

$$\vec{v}\cdot\vec{r} = r\dot{r}$$

$$\boxed{\ddot{x} = \frac{M}{r^3}\left[-x - xv^2 + 2r\dot{r}\dot{x}\right]}$$

$$\vec{a}\cdot\vec{v} = \Bigg[(\vec{v}\cdot\nabla)\Phi + (\vec{v}\cdot\nabla)(\vec{P}\cdot\vec{v}) + $$

$$ - \vec{v}\cdot\frac{\partial \vec{P}}{\partial t} - \left\{\left(\frac{\partial I}{\partial t}\vec{v}\right)\cdot\vec{v}\right\}$$

$$ + \Bigg\{ -(\vec{v}\cdot\nabla)\Phi + \frac{\partial \vec{P}}{\partial t}\cdot\vec{v} - (\vec{v}\cdot\nabla)(\vec{P}\cdot\vec{v})$$

$$ + \left\{\left(\frac{\partial I}{\partial t}\vec{v}\right)\cdot\vec{v}\right\} \Bigg\} v^2 \Bigg]$$

$$(\vec{v}\cdot\nabla)\vec{P} = v^x\frac{\partial \vec{P}}{\partial x} + v^y\frac{\partial \vec{P}}{\partial y} + v^z\frac{\partial \vec{P}}{\partial z}$$

$$\left[(\vec{v}\cdot\nabla)\vec{P}\right]\cdot\vec{v} = v^x\frac{\partial(\vec{P}\cdot\vec{v})}{\partial x} + v^y\frac{\partial(\vec{P}\cdot\vec{v})}{\partial y} + v^z\frac{\partial(\vec{P}\cdot\vec{v})}{\partial z}$$

$$(\vec{v}\cdot\nabla)\vec{P} = \Bigg[i\left(v^x\frac{\partial P^x}{\partial x} + v^y\frac{\partial P^x}{\partial y} + v^z\frac{\partial P^x}{\partial z}\right)$$

$$ + j\left(v^x\frac{\partial P^y}{\partial x} + v^y\frac{\partial P^y}{\partial y} + v^z\frac{\partial P^y}{\partial z}\right)$$

$$ + k\left(v^x\frac{\partial P^z}{\partial x} + v^y\frac{\partial P^z}{\partial y} + v^z\frac{\partial P^z}{\partial z}\right)$$

$$(\vec{v}\cdot\nabla)(T\vec{v}) =$$

$$
\begin{aligned}
i\Big[\; & v^x\frac{\partial}{\partial x}(T^{xx}v^x) + v^y\frac{\partial}{\partial y}(T^{xx}v^x) + v^z\frac{\partial}{\partial z}(T^{xx}v^x) \\
+\; & v^x\frac{\partial}{\partial x}(T^{xy}v^y) + v^y\frac{\partial}{\partial y}(T^{xy}v^y) + v^z\frac{\partial}{\partial z}(T^{xy}v^y) \\
+\; & v^x\frac{\partial}{\partial x}(T^{xz}v^z) + v^y\frac{\partial}{\partial y}(T^{xz}v^z) + v^z\frac{\partial}{\partial z}(T^{xz}v^z)\Big] \\
+\,j\Big[\; & v^x\frac{\partial}{\partial x}(T^{yx}v^x) + v^y\frac{\partial}{\partial y}(T^{yx}v^x) + v^z\frac{\partial}{\partial z}(T^{yx}v^x) \\
+\; & v^x\frac{\partial}{\partial x}(T^{yy}v^y) + v^y\frac{\partial}{\partial y}(T^{yy}v^y) + v^z\frac{\partial}{\partial z}(T^{yy}v^y) \\
+\; & v^x\frac{\partial}{\partial x}(T^{yz}v^z) + v^y\frac{\partial}{\partial y}(T^{yz}v^z) + v^z\frac{\partial}{\partial z}(T^{yz}v^z)\Big] \\
+\,K\Big[\; & v^x\frac{\partial}{\partial x}(T^{zx}v^x) + v^y\frac{\partial}{\partial y}(T^{zx}v^x) + v^z\frac{\partial}{\partial z}(T^{zx}v^x) \\
+\; & v^x\frac{\partial}{\partial x}(T^{zy}v^y) + v^y\frac{\partial}{\partial y}(T^{zy}v^y) + v^z\frac{\partial}{\partial z}(T^{zy}v^y) \\
+\; & v^x\frac{\partial}{\partial x}(T^{zz}v^z) + v^y\frac{\partial}{\partial y}(T^{zz}v^z) + v^z\frac{\partial}{\partial z}(T^{zz}v^z)\Big]
\end{aligned}
$$

$$\left\{(\vec{v}\cdot\nabla)\vec{P}\right\}\cdot\vec{v} = \left[\begin{aligned} & v^x\frac{\partial P^x}{\partial x}v^x + v^y\frac{\partial P^x}{\partial y}v^x + v^z\frac{\partial P^x}{\partial z}v^x \\ & + v^x\frac{\partial P^y}{\partial x}v^y + v^y\frac{\partial P^y}{\partial y}v^y + v^z\frac{\partial P^y}{\partial z}v^y \\ & + v^x\frac{\partial P^z}{\partial x}v^z + v^y\frac{\partial P^z}{\partial y}v^z + v^z\frac{\partial P^z}{\partial z}v^z \end{aligned}\right.$$

$$\left\{(\vec{v}\cdot\nabla)\vec{P}\right\}\cdot\vec{v} = \left(v^x\frac{\partial}{\partial x} + v^y\frac{\partial}{\partial y} + v^z\frac{\partial}{\partial z}\right)\left(P^x v^x + P^y v^y + P^z v^z\right)$$

$$\boxed{\left\{(\vec{v}\cdot\nabla)\vec{P}\right\}\cdot\vec{v} = (\vec{v}\cdot\nabla)(\vec{P}\cdot\vec{v})}$$

$$\underline{(\vec{v}\cdot\nabla)(T\vec{v})}$$

$$T\vec{v} = \left[\begin{aligned} & i\left(T^{xx}v^x + T^{xy}v^y + T^{xz}v^z\right) \\ & + j\left(T^{yx}v^x + T^{yy}v^y + T^{yz}v^z\right) \\ & + k\left(T^{zx}v^x + T^{zy}v^y + T^{zz}v^z\right) \end{aligned}\right]$$

$$\left[\left(\vec{v}\cdot\nabla\right)\left(T\vec{v}\right)\right]\cdot\vec{v} =$$

$$v^x v^x \frac{\partial}{\partial x}\left(T^{xx} v^x\right) + v^x v^y \frac{\partial}{\partial y}\left(T^{xx} v^x\right) + v^x v^z \frac{\partial}{\partial z}\left(T^{xx} v^x\right)$$

$$+ v^x v^x \frac{\partial}{\partial x}\left(T^{xy} v^y\right) + v^x v^y \frac{\partial}{\partial y}\left(T^{xy} v^y\right) + v^x v^z \frac{\partial}{\partial z}\left(T^{xy} v^y\right)$$

$$+ v^x v^x \frac{\partial}{\partial x}\left(T^{xz} v^z\right) + v^x v^y \frac{\partial}{\partial y}\left(T^{xz} v^z\right) + v^x v^z \frac{\partial}{\partial z}\left(T^{xz} v^z\right)$$

$$+ v^y v^x \frac{\partial}{\partial x}\left(T^{yx} v^x\right) + v^y v^y \frac{\partial}{\partial y}\left(T^{yx} v^x\right) + v^y v^z \frac{\partial}{\partial z}\left(T^{yx} v^x\right)$$

$$+ v^y v^x \frac{\partial}{\partial x}\left(T^{yy} v^y\right) + v^y v^y \frac{\partial}{\partial y}\left(T^{yy} v^y\right) + v^y v^z \frac{\partial}{\partial z}\left(T^{yy} v^y\right)$$

$$+ v^y v^x \frac{\partial}{\partial x}\left(T^{yz} v^z\right) + v^y v^y \frac{\partial}{\partial y}\left(T^{yz} v^z\right) + v^y v^z \frac{\partial}{\partial z}\left(T^{yz} v^z\right)$$

$$+ v^z v^x \frac{\partial}{\partial x}\left(T^{zx} v^x\right) + v^z v^y \frac{\partial}{\partial y}\left(T^{zx} v^x\right) + v^z v^z \frac{\partial}{\partial z}\left(T^{zx} v^x\right)$$

$$+ v^z v^x \frac{\partial}{\partial x}\left(T^{zy} v^y\right) + v^z v^y \frac{\partial}{\partial y}\left(T^{zy} v^y\right) + v^z v^z \frac{\partial}{\partial z}\left(T^{zy} v^y\right)$$

$$+ v^z v^x \frac{\partial}{\partial x}\left(T^{zz} v^z\right) + v^z v^y \frac{\partial}{\partial y}\left(T^{zz} v^z\right) + v^z v^z \frac{\partial}{\partial z}\left(T^{zz} v^z\right)$$

$$\left[(\vec{v}\cdot\nabla)(T\vec{v})\right]\cdot\vec{v} = (\vec{v}\cdot\nabla)\left\{(T\vec{v})\cdot\vec{v}\right\}$$

$$\vec{a}\cdot\vec{v} = \left[(\vec{v}\cdot\nabla)\Phi + (\vec{v}\cdot\nabla)(\vec{P}\cdot\vec{v}) - \frac{\partial\vec{P}}{\partial t}\cdot\vec{v} - \left(\frac{\partial T}{\partial t}\vec{v}\right)\cdot\vec{v}\right](1-v^2)$$

$$\boxed{\frac{\vec{a}\cdot\vec{v}}{1-v^2} = \left[(\vec{v}\cdot\nabla)\Phi + (\vec{v}\cdot\nabla)(\vec{P}\cdot\vec{v}) - \frac{\partial}{\partial t}(\vec{P}\cdot\vec{v}) - \frac{\partial}{\partial t}\left\{(T\vec{v})\cdot\vec{v}\right\}\right]}$$

$$\vec{a} = \Big| \nabla\left[\Phi + 2(\vec{P}\cdot\vec{v}) + (T\vec{v})\cdot\vec{v}\right] - \frac{\partial\vec{P}}{\partial t} - (\vec{v}\cdot\nabla)\vec{P} - \frac{\partial T}{\partial t}\vec{v} - (\vec{v}\cdot\nabla)(T\vec{v}) - \frac{\vec{a}\cdot\vec{v}}{1-v^2}\vec{v}$$

$$\boxed{\vec{a} + \frac{\vec{a}\cdot\vec{v}}{1-v^2}\,\vec{v} = \begin{aligned}[t] &\nabla\left[\Phi + 2(\vec{p}\cdot\vec{v}) + (T\vec{v})\cdot\vec{v}\right] \\ &- \frac{\partial\vec{p}}{\partial t} - (\vec{v}\cdot\nabla)\vec{p} \\ &- \frac{\partial T}{\partial t}\vec{v} - (\vec{v}\cdot\nabla)(T\vec{v}) \end{aligned}}$$

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T

$$\cancel{x' = \frac{dx}{dt}}$$

$$\left.\begin{aligned}
\ddot{x} &= \frac{x''}{t'^2} - \dot{x}\frac{t''}{t'^2} \\
\ddot{y} &= \frac{y''}{t'^2} - \dot{y}\frac{t''}{t'^2} \\
\ddot{z} &= \frac{z''}{t'^2} - \dot{z}\frac{t''}{t'^2}
\end{aligned}\right\}
\begin{aligned}
\dot{x} &= \frac{x'}{t'} \\
\dot{y} &= \frac{y'}{t'} \\
\dot{z} &= \frac{z'}{t'}
\end{aligned}$$

$$\dot{x}\ddot{x} + \dot{y}\ddot{y} + \dot{z}\ddot{z} = \boxed{\frac{x'x'' + y'y'' + z'z''}{t'^3} - (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)\frac{t''}{t'^2}}$$

$$\vec{a}\cdot\vec{v} = \cancel{-t't'' - v^2 t''}$$

$$\vec{a}\cdot\vec{v} = +\frac{t't''}{t'^3} - v^2\frac{t''}{t'^2}$$

$$\vec{a}\cdot\vec{v} = +(1 - v^2)\frac{t''}{t'^2}$$

$$\frac{\vec{a}\cdot\vec{v}}{1 - v^2} = \cancel{-}\frac{t''}{t'^2}$$

$$\ddot{x} = \frac{x''}{t'^2} - \dot{x}\frac{t''}{t'^2} \qquad \dot{x} = \frac{x'}{t'}$$

$$\ddot{y} = \frac{y''}{t'^2} - \dot{y}\frac{t''}{t'^2} \qquad \dot{y} = \frac{y'}{t'}$$

$$\ddot{z} = \frac{z''}{t'^2} - \dot{z}\frac{t''}{t'^2} \qquad \dot{z} = \frac{z'}{t'}$$

$$\dot{x}\ddot{x} + \dot{y}\ddot{y} + \dot{z}\ddot{z} = \frac{x'x'' + y'y'' + z'z''}{t'^3} - (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)\frac{t''}{t'^2}$$

$$\boxed{\vec{a}\cdot\vec{v} = \frac{x'x'' + y'y'' + z'z''}{t'^3} - v^2\frac{t''}{t'^2}}$$

$$t'^2 - x'^2 - y'^2 - z'^2 = 1$$

$$t't'' - x'x'' - y'y'' - z'z'' = 0$$

$$\boxed{x'x'' + y'y'' + z'z'' = t't''}$$

$$\vec{a}\cdot\vec{v} = \frac{tt''}{t'^3} - v^2\frac{t''}{t'^2}$$

$$\vec{a}\cdot\vec{v} = (1-v^2)\frac{t''}{t'^2}$$

$$\boxed{\frac{\vec{a}\cdot\vec{v}}{1-v^2} = \frac{t''}{t'^2}}$$

$$\vec{a} + \frac{\vec{a}\cdot\vec{v}}{1-v^2}\vec{v} = \left(\frac{x''}{t'^2}, \frac{y''}{t'^2}, \frac{z''}{t'^2}\right)$$

$$t'^2 - x'^2 - y'^2 - z'^2 = 1$$

$$t'^2 - t'^2\left(\frac{x'^2}{t'^2} - \frac{y'^2}{t'^2} - \frac{z'^2}{t'^2}\right) = 1$$

$$t'^2\left[1-v^2\right] = 1$$

$$\cancel{t} \quad \boxed{t' = \frac{1}{\sqrt{1-v^2}}}$$

$$\frac{\vec{a}\cdot\vec{v}}{(1-v^2)^2} = t''$$

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2$$

$$ds^2 = dt^2\left[1-v^2\right]$$

$$\left(\frac{dt}{ds}\right)^2 = \frac{1}{1-v^2} \qquad \boxed{t' = \frac{1}{\sqrt{1-v^2}}}$$

$$t'^2 = \frac{1}{1-v^2}$$

$$\frac{\vec{a}\cdot\vec{v}}{1-v^2} = t''(1-v^2)$$

$$\boxed{\frac{\vec{a}\cdot\vec{v}}{(1-v^2)^2} = t''}$$

$$t'' = \left[-t'^2 \frac{\partial \vec{P}}{\partial t}\cdot\vec{v} + t'^2\left\{(\vec{v}\cdot\nabla)\Phi\right\} - t'^2\left\{\left(\frac{\partial T}{\partial t}\vec{v}\right)\cdot\vec{v}\right\} + t'^2(\vec{v}\cdot\nabla)(\vec{P}\cdot\vec{v}) \right]$$

$$\frac{t''}{t'^2} = \left[-\frac{\partial \vec{P}}{\partial t}\cdot\vec{v} + \left\{(\vec{v}\cdot\nabla)\Phi\right\} - \left\{\left(\frac{\partial T}{\partial t}\vec{v}\right)\cdot\vec{v}\right\} + (\vec{v}\cdot\nabla)(\vec{P}\cdot\vec{v}) \right]$$

$$\boxed{\frac{\vec{a}\cdot\vec{v}}{1-v^2} = \left[-\frac{\partial \vec{P}}{\partial t}\cdot\vec{v} + \left\{(\vec{v}\cdot\nabla)\Phi\right\} - \left\{\left(\frac{\partial T}{\partial t}\vec{v}\right)\cdot\vec{v}\right\} + (\vec{v}\cdot\nabla)(\vec{P}\cdot\vec{v}) \right]}$$

$$\vec{a} + \frac{\vec{a}\cdot\vec{v}}{1-v^2}\vec{v} =$$

$$\left[\nabla\left[\Phi + 2(\vec{P}\cdot\vec{v}) + (T\vec{v})\cdot\vec{v}\right] - \frac{\partial\vec{P}}{\partial t} - (\vec{v}\cdot\nabla)\vec{P} - \frac{\partial T}{\partial t}\vec{v} - (\vec{v}\cdot\nabla)(T\vec{v}) \right]$$

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$$\frac{d}{dt}\left(\frac{\vec{v}}{\sqrt{1-v^2}}\right) = \frac{\vec{a}}{\sqrt{1-v^2}} + \frac{\vec{a}\cdot\vec{v}}{(1-v^2)^{3/2}}\vec{v}$$

$$\frac{d}{dt}\left(\frac{\vec{v}}{\sqrt{1-v^2}}\right) = \frac{1}{\sqrt{1-v^2}}\left[\vec{a} + \frac{\vec{a}\cdot\vec{v}}{1-v^2}\vec{v}\right]$$

$$\frac{d}{dt}\left[x', y', z'\right] = \frac{d}{dt}\left[\frac{\vec{v}}{\sqrt{1-v^2}}\right]$$

Caso del Campo generado por un punto masa M en movimiento arbitrario

$$\Phi = M \frac{1+V^2}{\sqrt{1-V^2}\,(r-\vec{r}\cdot\vec{V})} = \frac{M(-1+V^2+1)}{\sqrt{1-V^2}\,(r-\vec{r}\cdot\vec{V})}$$

$$\vec{P} = -\frac{2M\vec{V}}{\sqrt{1-V^2}\,(r-\vec{r}\cdot\vec{V})}$$

$$T^{\alpha\beta} = \frac{2MV^\alpha V^\beta}{\sqrt{1-V^2}\,(r-\vec{r}\cdot\vec{V})} + \frac{M\delta^{\alpha\beta}\sqrt{1-V^2}}{(r-\vec{r}\cdot\vec{V})}$$

$$\vec{P}\cdot\vec{v} = -\frac{2M(\vec{V}\cdot\vec{v})}{\sqrt{(1-V^2)}\,(r-\vec{r}\cdot\vec{V})}$$

$$T^{\alpha\beta}v^\beta = \frac{2MV^\alpha(\vec{V}\cdot\vec{v})}{\sqrt{1-V^2}\,(r-\vec{r}\cdot\vec{V})} + \frac{Mv^\alpha\sqrt{1-V^2}}{(r-\vec{r}\cdot\vec{V})}$$

$$\Phi = -\frac{M\sqrt{1-V^2}}{(r-\vec{r}\cdot\vec{V})} + \frac{M}{\sqrt{1-V^2}\,(r-\vec{r}\cdot\vec{V})}$$

$$(T^{\alpha\beta}v^\beta)v^\alpha = \Bigg[\frac{2M(\vec{V}\cdot\vec{v})^2}{\sqrt{1-V^2}\,(r-\vec{r}\cdot\vec{V})} + \cancel{M\vec{V}\cdot\vec{v}^2} + \frac{Mv^2\sqrt{1-V^2}}{(r-\vec{r}\cdot\vec{V})}$$

$$T^{\alpha\beta} v^{\beta} = T\vec{v} = -(\vec{P}\cdot\vec{v})\vec{V} + (1-V^4)\Phi\,\vec{v}$$

$$T\vec{v} = -(\vec{P}\cdot\vec{v})\vec{V} + (1-V^4)\Phi\,\vec{v}$$

$$(T\vec{v})\cdot\vec{v} = -(\vec{P}\cdot\vec{v})(\vec{V}\cdot\vec{v}) + (1-V^4)\Phi\, v^2$$

Campo Central

$$\boxed{\vec{a} = -\frac{M\vec{r}}{r^3}(1+v^2) + \frac{2M(\vec{r}\cdot\vec{v})\,\vec{v}}{r^3}}$$ Campo Central

$$\vec{a}\cdot\vec{v} = -\frac{M(\vec{r}\cdot\vec{v})}{r^3}(1+v^2) + \frac{2M(\vec{r}\cdot\vec{v})}{r^3}v^2$$

$$\vec{a}\cdot\vec{v} = -\frac{M(\vec{r}\cdot\vec{v})}{r^3} + \frac{M(\vec{r}\cdot\vec{v})v^2}{r^3}$$

$$\vec{a}\cdot\vec{v} = -\frac{M(\vec{r}\cdot\vec{v})}{r^3}(1-v^2)$$

$$\boxed{\frac{\vec{a}\cdot\vec{v}}{1-v^2} = -\frac{M(\vec{r}\cdot\vec{v})}{r^3}}$$ Campo Central

$$\boxed{\vec{a} + \frac{\vec{a}\cdot\vec{v}}{1-v^2}\vec{v} = -\frac{M\vec{r}}{r^3}(1+v^2) + \frac{M(\vec{r}\cdot\vec{v})}{r^3}\vec{v}}$$

$$\vec{a} = \Big[\nabla\left[\Phi + 2(\vec{P}\cdot\vec{v}) + (T\vec{v})\cdot\vec{v} \right] - \frac{\partial \vec{P}}{\partial t} - (\vec{v}\cdot\nabla)\vec{P} - \frac{\partial T}{\partial t}\vec{v} - \{(\vec{v}\cdot\nabla)T\}\vec{v} + \left[-(\vec{v}\cdot\nabla)\Phi + \frac{\partial \vec{P}}{\partial t}\cdot\vec{v} - \{(\vec{v}\cdot\nabla)\vec{P}\}\cdot\vec{v} + \left(\frac{\partial T}{\partial t}\vec{v}\right)\cdot\vec{v} \right]\vec{v} \Big]$$

$$\frac{d\vec{P}}{dt} = \frac{\partial \vec{P}}{\partial t} + (\vec{v}\cdot\nabla)\vec{P} ;$$

$$\frac{\partial T}{\partial t}\vec{v} + \{(\vec{v}\cdot\nabla)T\}\vec{v} = \frac{dT}{dt}\vec{v} .$$

$$\vec{a} = \Big[\nabla\left[\Phi + 2(\vec{P}\cdot\vec{v}) + (T\vec{v})\cdot\vec{v} \right] - \frac{d\vec{P}}{dt} - \left(\frac{dT}{dt}\vec{v}\right) + \vec{v}\Big[-(\vec{v}\cdot\nabla)\Phi - 2(\vec{v}\cdot\nabla)(\vec{P}\cdot\vec{v}) - \{(\vec{v}\cdot\nabla)T\}\vec{v}\cdot\vec{v} + 2\{(\vec{v}\cdot\nabla)\vec{P}\}\cdot\vec{v} + \{\{(\vec{v}\cdot\nabla)T\}\vec{v}\}\cdot\vec{v} + \frac{\partial \vec{P}}{\partial t}\cdot\vec{v} - \{(\vec{v}\cdot\nabla)\vec{P}\}\cdot\vec{v} + \left(\frac{\partial T}{\partial t}\vec{v}\right)\cdot\vec{v}$$

$$\vec{a} = \left[\begin{aligned} &\nabla\left[\Phi + 2(\vec{P}\cdot\vec{v}) + (T\vec{v})\cdot\vec{v}\right] \\ &- \frac{d\vec{P}}{dt} - \left(\frac{dT}{dt}\vec{v}\right) \\ &+ \vec{v}\left\{ \begin{aligned} &-(\vec{v}\cdot\nabla)\left[\Phi + 2(\vec{P}\cdot\vec{v}) + (T\vec{v})\cdot\vec{v}\right] \\ &+ \vec{v}\cdot\frac{d\vec{P}}{dt} + \left(\frac{dT}{dt}\vec{v}\right)\cdot\vec{v} \end{aligned} \right\} \end{aligned} \right]$$

$$\vec{a} = \left[\begin{aligned} &\nabla\left[\Phi + 2(\vec{P}\cdot\vec{v}) + (T\vec{v})\cdot\vec{v}\right] \\ &- \frac{d\vec{P}}{dt} - \left(\frac{dT}{dt}\vec{v}\right) \\ &+ \vec{v}\left[\vec{v}\cdot\left(\begin{aligned} &\nabla\left[\Phi + 2(\vec{P}\cdot\vec{v}) + (T\vec{v})\cdot\vec{v}\right] \\ &- \frac{d\vec{P}}{dt} - \left(\frac{dT}{dt}\vec{v}\right) \end{aligned} \right) \right] \end{aligned} \right]$$

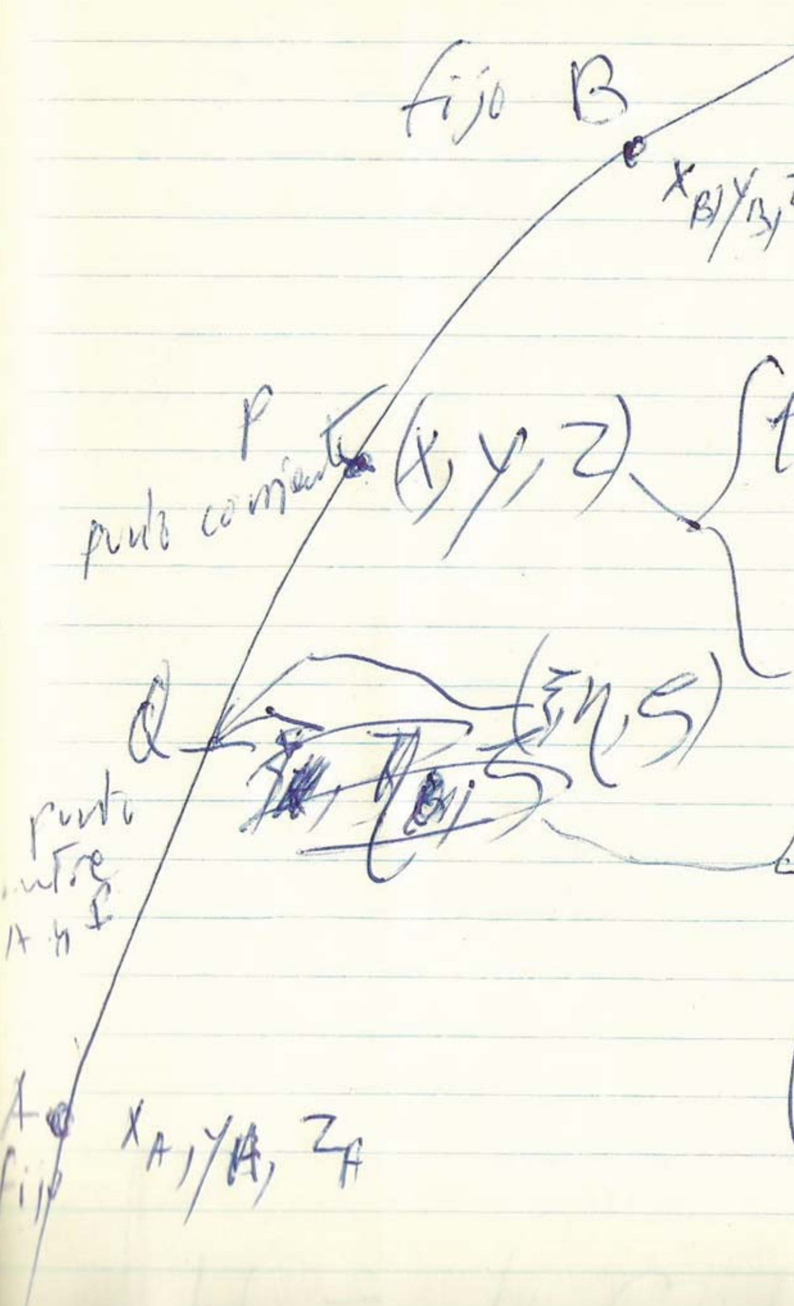
$$\psi = \Phi + 2(\vec{P}\cdot\vec{v}) + (T\vec{v})\cdot\vec{v}$$

$$\vec{a} = \left[\nabla\psi - \frac{d\vec{P}}{dt} - \frac{dT}{dt}\vec{v} - \vec{v}\cdot\left\{\vec{v}\cdot\left\{\nabla\psi - \frac{d\vec{P}}{dt} - \frac{dT}{dt}\vec{v}\right\}\right\}\right]$$

$$\vec{\Omega} = \nabla\psi - \frac{d\vec{P}}{dt} - \frac{dT}{dt}\vec{v}$$

$$\boxed{\vec{a} = \vec{\Omega} - (\vec{v}\cdot\vec{\Omega})\vec{v}}$$

ESPACIO FISICO.



trayectoria de una partícula exploradora

t instante en que la partícula pasa por P.

τ instante en que la partícula pasa por Q

Definiciones:

$$\boxed{G = \int_A^P \left[\Phi + 2\left(\vec{P} \cdot \vec{v}\right) + \left(T\vec{v}\right) \cdot \vec{v} \right] d\tau}$$

$$\boxed{G_1 = \int_A^P \Phi \, d\tau}$$

$$\boxed{G_2 = 2\int_A^P \left(\vec{P} \cdot \vec{v}\right) d\tau}$$

$$\boxed{G_3 = \int_A^P \left\{\left(T\vec{v}\right) \cdot \vec{v}\right\} d\tau}$$

$$\boxed{H = \int_A^B G \, dt}$$

~~to G_1 A~~

$$\boxed{G_1 = \int_A^P \Phi \, d\tau}$$

$$\boxed{\delta G_1 = \int_A^P \left(\nabla \Phi \cdot \delta\vec{\rho}\right) d\tau}$$

$$\boxed{G_2 = 2\int_A^P \left(\vec{P} \cdot \vec{v}\right) d\tau}$$

$$\delta G_2 = 2\int_A^P \left[\left\{(\delta\vec{\rho}\cdot\nabla)\vec{P}\right\}\cdot\vec{v}\, d\tau + \left\{\vec{P}\cdot\left(\frac{d}{d\tau}\delta\vec{\rho}\right)\right\} d\tau \right]$$

$$\delta G_2 = 2\,\vec{P}\cdot\delta\vec{\rho}\,\Big|_A^P + 2\int_A^P \left\{ \left\{(\delta\vec{\rho}\cdot\nabla)\vec{P}\right\}\cdot\vec{v}\, d\tau - \left\{\frac{d\vec{P}}{d\tau}\cdot\delta\vec{\rho}\right\} d\tau \right\}$$

$$\boxed{\delta G_2 = 2\,\vec{P}\cdot\delta\vec{\rho} + 2\int_A^P \left\{ \left((\delta\vec{\rho}\cdot\nabla)\vec{P}\right)\vec{v}\,d\tau - \left(\frac{d\vec{P}}{d\tau}\cdot\delta\vec{\rho}\right)d\tau \right\}}$$